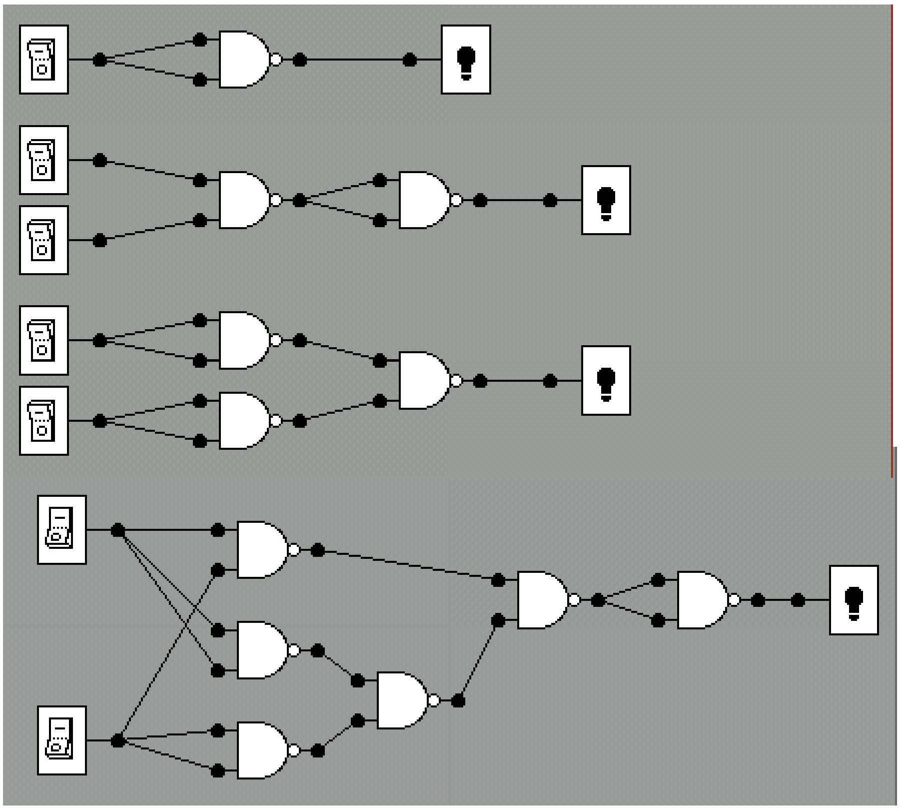

The circuits at the left demonstrate how to build an AND gate, an OR gate, a NOT gate, and an XOR gate, using only NAND gates.

(1) Which circuit represents which gate?

(2) The bottom circuit is quite complex. Is there a simpler circuit which is equivalent, still using only NAND gates?

(3) For the first three circuits, write an equivalent Boolean expression using only NOT and AND as connectors.

(4) Consider this Boolean Algebra expression: **(not A and not B) xor (A or B)**

(4a) Use a truth table to analyse this expression:

A	B	**not** A	**not** B	not A **and** not B	A **or** B	(not A and not B) **xor** (A or B)
0	0					
0	1					
1	0					
1	1					

(4b) Draw a circuit which is equivalent to the expression. Test the results of your truth table by using the circuit simulator.

(4c) The teacher will demonstrate a Boolean Algebra simplification which proves that the truth table was correct.

(5) Using Boolean Algebra rules, simplify each expression to a single operation (gate).

not (not A or not B)   (A or B) or (A nor B)   (A and not B) or (A and B)

(6) An optical sensor is attached to a door, registering the door as CLOSED (1) or OPEN (0) . A thermal sensor measures the heat of the room, registering TOO HOT (1) or OKAY (0) . An audio sensor measures noise in the room, registering NOISE (1) or QUIET (0) . A switch is available for the occupant to switch on the air conditioning (cooling). When the switch is ON (1), the air-conditioning will be on if the TOO HOT (1) sensor is on and the door is CLOSED (1). The air conditioner cannot be turned on when the door is OPEN(0) or the temperature is OKAY (0). Which of the following expressions correctly express the logic of this system? (there may be more than one correct)

(6a)
```
If  DOOR = OPEN  or  TEMPERATURE = OKAY  or  SWITCH = OFF  then
          TurnOffCooler
   else
          TurnOnCooler
end if
```

(6b) COOLER = ((DOOR NOR TEMPERATURE) NOR SWITCH)

(6c) COOLER = DOOR and SWITCH and TEMPERATURE

(6d) Construct a circuit to correctly control the air conditioner, using only NAND gates.

(7) A circuit is required which has three inputs. It produces 0 if all three inputs are 1, otherwise it produces 1.

(7a) One possibility for this circuit is based on the following expression:

```
If  (NOT A)  or  (NOT B)  or  (NOT C)  then
     Result = 1
  else
     Result = 0
end if
```

Draw and test the corresponding circuit.

(7b) Another possibility is based on the following logic:

```
If  A = 1  AND  B = 1  AND  C = 1  then
     Result = 0
  else
     Result = 1
end if
```

(8) The **Associative Law** states that **(A or B) or C** is equivalent to **A or (B or C)**.
In other words, in a string of ORs, it is irrelevant which one is evaluated first.
The same is true for AND: **(A and B) and C** is equivalent to **A and (B and C)**

For each of the gates XOR , NAND, and NOR, decide whether the Associative Law holds.

(9) The **Distributive Law** works as follows: **A and (B or C) == (A and B) or (A and C)**
Since AND is similar to multiplication in math, and OR is similar to addition, we would expect the distributive law to be correct. But there is no guarantee.

(9a) Build and compare two circuits to check whether AND is distributive over OR.

(9b) Build and compare two circuits to check whether the following are equivalent:
A or (B and C) == (A or B) and (A or C)

(10) DeMorgan's laws state that the following are equivalent:
not A and not B == not(A or B) **not A or not B == not(A and B)**

(10a) Construct circuits to show that DeMorgan's laws are correct.

(10b) Using any laws from above, construct a SIMPLER circuit equivalent to: **(not A and not B) or (A and B)**