Algorithms from IB Examinations

Previous examination paper questions, even those that apply to an earlier syllabus, can be a valuable resource because they can be used:

- ☒ to illustrate algorithms that relate either directly to the syllabus details
- ☒ to strengthen the problem-solving skills referred to in the introduction to Section B
- ☒ as a source of data and information for school-based tests.

This section consists of examples of questions that required candidates to trace and/or construct algorithms and their solutions. Some are complete questions, and others are parts of questions; they both refer to algorithm tracing or construction. In some cases solutions have been provided because they include an algorithm as part of the answer.

All questions and solutions have been rewritten in PURE.

The questions are categorized according to year, level, paper, question number, the action on the algorithm required, and the content area of the question. At the time of compilation the M98 papers had not yet been taken by candidates.

No.	Year	Level &	Qu.	Action on Algorithm	Content of Question
1	M96	SL P1	1	Trace	Summing and Averaging
2	M96	SL P1	3	Trace	Counting Loops
3	M96	SL P2	1	Trace and construct	Determining Frequencies
4	M96	SL P2	5	Construct	**If-then-else** Statements
5	N96	SL P1	1	Trace	Dividing and Truncating to Manipulate Single-decimal Digits
6	N96	SL P1	3	Trace and construct	Nonsense with Absolute Value
7	N96	SL P2	5	Trace and construct	Calculating Typing Fees
8	M97	SL P1	1	Trace and analyse	Golf Scores in Arrays
9	M97	SL P1	3	Trace	Summing Loop
10	M97	SL P2	1	Construct	One-dimensional Array Manipulation (Hashing)
11	N97	SL P1	1	Trace	Finding the Largest Value in an Array
12	N97	SL P1	3	Trace and modify	Binary Search
13	N97	SL P2	5	Construct	Simulation to Count Fish
14	M96	HL P1	10	Trace and construct	Highest Values in a Two-dimensional Array
15	M96	HL P2	4	Trace and construct	Fibonnaci Sequence (recursive)
16	N96	HL P2	1	Trace and construct	Binary Search
17	N96	HL P2	7	Trace and modify	Binary Tree
18	N97	HL P1	10	Trace and construct	Summing in a Two-dimensional Array
19	N97	HL P2	1	Trace and construct	Random Numbers

Summing and Averaging (M96 SL P1 Q1)

The following algorithm fragment has been designed to analyse the temperatures at a tourist resort.

```
COUNT, TOTAL, TEMP, BIG, AVERAGE  // variables

 1    COUNT = 0
 2    TOTAL = 0
 3    input TEMP
 4    BIG  = TEMP
 5    loop while not(TEMP = 0)
 6          TOTAL = TOTAL + TEMP
 7          COUNT = COUNT + 1
 8          input TEMP
 9          if  TEMP > BIG then
10                BIG = TEMP
11          endif
12    end loop
13    AVERAGE = TOTAL / COUNT
14    output AVERAGE, BIG
```

Copy and complete the following trace table for the data: 15, 7, 23, 9, 0

Line	COUNT	TOTAL	TEMP	BIG	TEMP # 0	**output**
1	0					
2		0				
3			15			
4				15		
5					true	
6						

Counting Loops (M96 SL P1 Q3)

(a) Consider the algorithm fragment below.

```
X = 1
loop while X < 6
      Y = 1
      loop while Y < 5
            Y = Y + 1
      end loop
      output "The product of X and Y is ", X * Y
      output "The values of X and Y are ", X, Y
      X = X + 1
end loop
```

Complete the trace through the fragment. The first two lines are provided and your output should be similar to that below:

The product of X and Y is   5
The values of X and Y are   1 5

.
.
.

(b) Consider the algorithm fragment below.

```
X = 1
loop while X < 6
      Y = 1
      loop while Y < 6
            output "The product of X and Y is ", X * Y
            Y <-- Y + 2
      end loop
      output "The values of X and Y are ", X, Y
      X = X + 2
end loop
```

Complete the trace through the fragment. The first line is provided and your output should be similar to that below:

The product of X and Y is 1

.
.
.

Determining Frequencies (M96 SL P2 Q1)

In your school, you must determine the number of students taking various examinations. As part of the input process, a student enters the code of a subject into a computer. For example, 21 could represent computer science. These are stored in an array and the frequency (number of times) with which each code occurs is to be determined.

(a) Consider the following subject code data:

DATA: 21 14 25 23 21 21 25 14 16 16 17 21 25 17 21 23 14 25 17

Copy and complete the tables below to show the unique subject codes and their corresponding frequency. Use the subject code data above.

CODES
21

FREQS

(b) Consider the following variables:

`CODES` single-dimension array of integer values which will eventually contain the subject codes without duplications.
(Initialized for a maximum of 750 values.)

`FREQS` single-dimension array of integer values which will eventually contain the frequency of the subject codes.
(Initialized for a maximum of 750 values.)

`SIZE` Integer variable that indicates the current number of valid entries in `CODES` and `FREQS`.

`CODE` Integer variable that contains an input subject code.

`FOUND` Boolean variable that indicates if the current value in `CODE` is already stored in `CODES`.

`POSITION` Integer variable that indicates the position that `CODE` should be placed in the `CODES` array.

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Determining Frequencies (M96 SL P2 Q1) (cont.)

The algorithm below (FREQUENCIES) uses these variables to compute the subject code frequencies.

```
SIZE = 0

input CODE    //  the input stream is terminated by -99

loop while (SIZE <= 750) and (CODE <> -99)

      SEARCH(CODES, SIZE, CODE, FOUND, POSITION)

             /*   CODES, FOUND and POSITION are
                  pass-by-reference parameters;
                  all others are pass-by-value    */

      if not FOUND then
            . . . .
            . . . .
            . . . .
```

(i) The subalgorithm SEARCH tests if the current subject code (stored in CODE) is already in array CODES. Explain how the parameters POSITION and FOUND will be effected by SEARCH if:

- CODE is already in array CODES
- CODE is not already in array CODES.

(ii) Construct subalgorithm SEARCH.

(iii) Explain how the contents of FREQS and CODES will be updated by the algorithm FREQUENCIES if:

- CODE is already in array CODES
- CODE is not already in array CODES.

(iv) Complete the algorithm FREQUENCIES.

(This question continues on the following page)

3 Determining Frequencies (M96 SL P2 Q1) (cont.)

A possible solution to (b) (ii)

```
procedure SEARCH(CODES integer array, SIZE integer,
                 CODE integer, FOUND boolean,
                 POSITION integer)

    FOUND = false
    X = 0
    loop while (X < SIZE) and not FOUND
        X = X + 1
        if CODES[X] = CODE then
            FOUND = true
        endif
    end loop
    POSITION = X
endprocedure SEARCH
```

A possible solution to (b) (iv)

```
    SIZE = 0
    input CODE    //  the input stream is terminated by -99
    while (SIZE <= 750) and (CODE <> -99) do
        SEARCH(CODES, SIZE, CODE, FOUND, POSITION)
            //  CODES, FOUND and POSITION are
            //  pass-by-reference parameters; all others
            //  are pass-by-value

        if not FOUND then
            SIZE = SIZE + 1
            CODES[SIZE] = CODE
            FREQS[SIZE] = 1
        else
            FREQS[POSITION] = FREQS[POSITION] + 1
        end if
    endwhile
```

If-then-else Statements (M96 SL P2 Q5)

When evaluating the Boolean operators, **and** and **or**, in some circumstances the evaluation of the entire Boolean expression can be determined by the value of the first operand.

For example, the statement "S1 **and** S2" is false if S1 is false. Similarly, "S1 **or** S2" is true if S1 is true. In both of these cases, the value of S2 is irrelevant.

Rewrite the following algorithm fragments to use this strategy, testing only one operand at a time (e.g. **if** S1 **then**, **if not** S1 **then**).

(a) Implement the statement below. No Boolean operators are necessary.

```
if S1 or S2 then
     ACTION1
else
     ACTION2
endif
```

(b) Implement the statement below. The Boolean operator **not** is required.

```
if S1 and S2 then
     ACTION1
else
     ACTION2
endif
```

A possible solution for (a):

```
if S1 then
     ACTION1
else
     if S2 then
          ACTION1
     else
          ACTION2
     endif
endif
```

Two possible solutions for part (b):

```
if S1 then
     if S2 then
          ACTION1
     else
          ACTION2
     endif
else
     ACTION2
endif
```

```
if not S1 then
     ACTION2
else
     if not S2 then
          ACTION2
     else
          ACTION1
     endif
endif
```

5 Dividing and Truncating to Manipulate Single-decimal Digits (N96 SL P1 Q1)

Consider the procedure below.

```
        input NUMBER   // assume 123 is entered

1       NEWVALUE = 0
2       loop while NUMBER > 0
3            DIGIT = NUMBER - truncate(NUMBER / 10) * 10
4            NEWVALUE = NEWVALUE * 10 + DIGIT
5            NUMBER = truncate(NUMBER / 10)
6       end loop
7       output NEWVALUE
```

Remember that **truncate**(769.84) returns 769 , rounding DOWN to the nearest integer.

(a) Copy and complete the following trace table for the call ALTER(123).

Line	NUMBER	NEWVALUE	NUMBER > 0	DIGIT	**output**
	123				
1		0			
2			true		
3				3	

(b) Describe the purpose of the procedure ALTER.

(c) State the output for NUMBER = -123

(d) Explain the difference if ALTER used a pass-by-reference parameter rather than a pass-by-value parameter.

Nonsense with Absolute Value (N96 SL P1 Q3)

Consider the algorithm fragment below. Remember that **abs**(- 23.4) returns +23.4 and **abs**(1051.0) returns +1051.0.

```
input NUMBER1
input  NUMBER2

if NUMBER1 >= 0.0 then
      if NUMBER1 < 1000.0 then
            NUMBER2 <-- 2 * NUMBER1
            if NUMBER1 <= 500.0 then
                  NUMBER1 <-- NUMBER1 / 10.0
            endif
      else
            NUMBER2 <-- 3 * NUMBER1
      endif
else
      NUMBER2 <-- abs(NUMBER1)
endif
```

(a) State the values of `NUMBER1` and `NUMBER2` after this algorithm fragment is evaluated, given that the initial value of `NUMBER1` is:

(i) 381.5

(ii) -21.0

(iii) 1200.0

(b) Complete the following algorithm for the function **abs**.

```
function abs(val NUMBER real)
result  real
      declare  ANSWER  real

      . . . . . . .
      . . . . . . .
      . . . . . . .

      return ANSWER
endfunction abs
```

Calculating Typing Fees (N96 SL P2 Q5)

In order to earn extra money, a student types extended essays for a fee. The amount charged depends on the number of pages in the document. The student charges:

- ☒ 5.00 (of some monetary unit) minimum fee for one to three pages
- ☒ 1.50 per page for each page over three pages
- ☒ an additional 3.75 if the number of pages exceeds 10.

Assuming that 200 words fit on a single typed page, a 1300 word extended essay would produce a fee of 11.00. That is, 1300 + 200 = 6.5 actual pages, for which the student charges 7 whole pages. The calculation is 5.00 (for the first 3 pages) + 1.50 x 4 pages (7 - 3) to produce a fee of 11.00.

(a) Calculate the fees, showing all working, for the following extended essays of length:

(i) 1000 words

(ii) 2425 words.

(b) Construct the algorithm fragment which a student can use to calculate the fee.

The algorithm fragment must prompt the student to give the number of words in the extended essay. The desired output will be:

- ☒ actual number of pages
- ☒ whole number of pages
- ☒ typing fee.

Remember that `truncate`(769.84) returns 769.

The output should be clearly labelled and all variables defined.

Answers: (a)(i) 8.00 (a)(ii) 23.75

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Calculate Typing Fees (N96 SL P2 Q5) (cont.)

A possible solution for (b):

```
input NUMWORDS
TYPING = NUMWORDS / 200
if truncate(TYPING) = TYPING then
      PAGES = TYPING
else
      PAGES = truncate(TYPING) + 1
end if

FEE <-- 5
if PAGES > 3 then
      FEE <-- FEE +(PAGES - 3) * 1.5
end if
if PAGES > 10 then
      FEE <-- FEE + 3.75
end if

output "The actual number of pages is ", TYPING
output "A number of pages to be charged for is ", PAGES
output "The amount to be paid is ", FEE
```

Golf Scores in Arrays (M97 SL P1 Q1)

The best weekly scores of golfers at a golf club are stored in two arrays as follows:

NAME		SCORE
Jenkins	[1]	82
Zendra	[2]	77
Lirmin	[3]	78
Jenkins	[4]	76
Furniss	[5]	81
Jenkins	[6]	77
	[7]	-1

The array subscripts indicate the week of the score. For example, the best score at the club in week 3 was 78 which was made by Lirmin.

Consider the algorithm below.

```
LOC = 0
SUM = 0
NUMBER = 0
output "Enter a golfer's name"
input PERSON
loop while (LOC <> 53) and (SCORE[LOC] <> -1) do
      if NAME[LOC] = PERSON then
            SUM = SUM + SCORE[LOC]
            NUMBER = NUMBER + 1
      end if
      LOC = LOC + 1
end loop
ANSWER = SUM / NUMBER
output ANSWER
```

(a) State what should be the dimension of the arrays.

(b) State the sentinel value and in what array it is to be found.

(c) The variable ANSWER has not been declared. State its data type and justify your answer.

(d) Trace the algorithm with the input data "Jenkins" for the array entries given.

(e) Describe the purpose of the algorithm.

(f) Some inputs will cause the algorithm to fail. Explain when this will happen and give an outline solution to stop the error.

Summing Loop (M97 SL P1 Q3)

The following algorithms are designed to calculate the sum (total) of a series of integers:

```
procedure SUM1
      declare  SUM, VALUE  integer

      SUM = 0
      input VALUE
      loop while VALUE > 0
            SUM = SUM + VALUE
            input VALUE
      end loop
      output SUM
endprocedure SUM1

procedure SUM2
      declare  SUM, VALUE  integer

      SUM = 0
      input VALUE
      loop
            SUM = SUM + VALUE
            input VALUE
      while VALUE >= 0
      output SUM
endprocedure SUM2


procedure SUM3
      declare  SUM, VALUE, LENGTH, COUNT  integer

      input LENGTH
      SUM = 0
      loop COUNT from 1 to LENGTH
            input VALUE
            SUM = SUM + VALUE
      end loop
      output SUM
endprocedure SUM3
```

For **each** algorithm (SUM1, SUM2 and SUM3):

(a) Explain what kind of integers (negative and/or positive) the loops are designed to accept and what effect they have on the loops.

(b) State how many times the loop is executed.

Binary Search (N97 SL P1 Q3)

The following procedure searches for a number.

```
procedure SEARCH(val MARKS integer array, val MAXLENGTH integer,
                 val NUMBER integer)

   //  Assume that the array MARKS contains MAXLENGTH elements

      FOUND = false
      LOWER = 0
      LENGTH = MAXLENGTH + 1
      loop
            MIDDLE = LOWER + (LENGTH - LOWER) div 2
            if NUMBER = MARKS[MIDDLE] then
                  FOUND = true
            else
                  if NUMBER < MARKS[MIDDLE]  then
                        LENGTH = MIDDLE
                  else
                        LOWER = MIDDLE
                  endif
            endif
      while not FOUND

      if FOUND then
            output MIDDLE
      else
            output "Not found"
      endif
endprocedure SEARCH
```

(a) Trace the call SEARCH(GRADES, 11, 24) by means of a trace table. Show the changes to the variables.

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
GRADES	1	3	7	9	12	13	15	20	24	26	28

(b) State the output of the call SEARCH(GRADES, 7, 24).

(c) There is a problem when searching for the number 21.

(i) Identify the problem.

(ii) State what can be added to the algorithm in order to solve this problem and indicate where it must be added.

Simulation to Count Fish (N97 SL P2 Q5)

A computer is used to monitor the number of fish passing under a sensor in a lake. The sensor has a timing device which sends the character "T" to the processor every minute. If a fish is detected, the sensor sends the character "F" to the processor. At the end of the timing period, the "T" is immediately followed by an "E".

A typical data stream could be:

T F T T T F F T F F F F T F T E

This means that the first fish detected was between the first and second minute. Two fish passed under the sensor between the fourth and fifth minute, etc.

Construct an algorithm that inputs the data from the sensor and produces the following output:

- ☒ the number of minutes that the survey was in operation
- ☒ the total number of fish that passed under the sensor
- ☒ the largest number of fish that passed under the sensor in any one-minute interval.

The data that would be the output, for the above example, would be:

- ☒ the survey lasted 7 minutes
- ☒ a total of 8 fish passed under the sensor
- ☒ the largest number of fish in a one-minute time interval was 4.

(The pseudo-code instruction that reads a character from the `SENSOR` into the character variable `SIGNAL` is:

```
input(SENSOR, SIGNAL)
```

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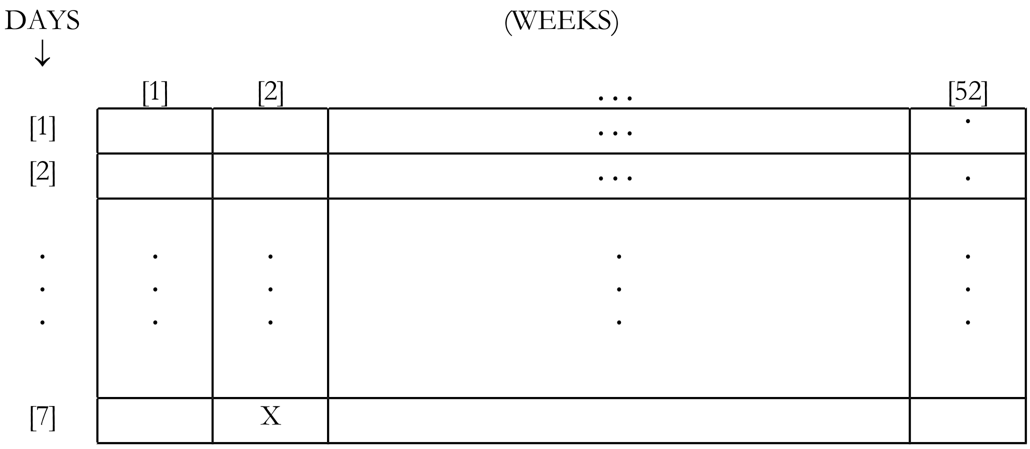
A possible solution:

```
main FISH

      FISHCOUNT = 0
      MINUTES = 0
      MAXFISH = 0
      THISFISH = 0

      input (SENSOR, SIGNAL)

      loop while SIGNAL <> "E"
            if SIGNAL = "F" then
                  FISHCOUNT = FISHCOUNT +1
                  THISFISH = THISFISH + 1
            else
                  if SIGNAL = "T" then
                        MINUTES = MINUTES + 1
                        if THISFISH > MAXFISH then
                              MAXFISH = THISFISH
                        endif
                        THISFISH = 0
                  endif
            endif
            input (SENSOR, SIGNAL)
      end loop
      output "The survey lasted ", MINUTES, " minutes"
      output "A total of ", FISHCOUNT, " fish passed under the
               sensor"
      output "The Largest number of fish in a one-minute time
               interval was ", MAXFISH
endmain FISH
```

Highest Values in a Two-dimensional Array (M96 HL P1 Q10)

The two-dimensional array of real values `TEMPS` outlined below contains the maximum daily temperatures over a period of one year.



The cell indicated by the X, `TEMPS (7, 2)`, would store the maximum temperature for Saturday in week 2.

By referring to this array, construct an algorithm that computes the highest and lowest maximum daily temperatures of the year (that is, the largest and smallest values in the array).

In addition to the temperature, the algorithm should clearly output the week of the year and the day of the week (specifying Sunday, Monday, etc.) on which these extreme temperatures occurred. (Assume that day 1 is Sunday, day 2 is Monday, etc.)

Define all variables used. (You may assume that valid real temperatures are already in the array `TEMPS.`)

14 Highest Values in a Two-dimensional Array (M96 HL P1 Q10) (cont.)

A possible solution:

```
HIGH = TEMPS[1,1]
LOW = TEMPS[1,1]
LWEEK = 1
LDAY = 1
HWEEK = 1
HDAY = 1
loop  X from 1 to 7
      loop  Y from 1 to 52 do
            if TEMPS[X,Y] > HIGH then
                  HIGH = TEMPS[X,Y]
                  HWEEK = Y
                  HDAY = X
            end if
            if TEMPS[X,Y] < LOW then
                  LOW = TEMPS[X,Y]
                  LWEEK = Y
                  LDAY = X
            end if
      end loop
end loop

output " The highest temperature for the year is ", HIGH,
            " and it occurred in week ", HWEEK, " on a "
if HDAY = 1 then
      output "Sunday"
else if HDAY = 2 then
      output "Monday"
else if HDAY = 3 then
            .
            .
else if HDAY = 7 then
      output "Saturday"
end if

output " The lowest temperature for the year is ", LOW,
            " and it occurred in week ", LWEEK, " on a "

if LDAY = 1 then
      output "Sunday"
else if LDAY = 2 then
      output "Monday"
else if LDAY = 3 then
            .
            .
else if LDAY = 7 then
      output "Saturday"
end if
```

15 Fibonnaci Sequence (recursive) (M96 HL P2 Q4)

The algorithm fragment below generates values that are called the Fibonnaci sequence.

```
P1 = 1
NEXT = 1
output P1
output NEXT
loop X from 1 to 10
      P2 = P1
      P1 = NEXT
      NEXT = P2 + NEXT
      output NEXT
end loop
```

(a) What are the Fibonnaci numbers generated by this algorithm?

(b) Given the algorithm below (FIBSEQ), construct the recursive subalgorithm for the function FIBONNACI (used near the end of the algorithm) that will generate the N^{th} Fibonnaci number (when N >= 2).

Define and describe all parameters and variables used in the recursive algorithm.

Hint: the nth Fibonnaci number [or FIBONNACI(N)] is given by the $(N - 1)^{th}$ + the $(N - 2)^{th}$ Fibonnaci number, for N > 2.

```
main FIBSEQ
      declare  N, FIB  integer

      output "Enter the number, n, greater than 2, for
                the nth Fibonnaci number to be generated."
      input N
      loop while N <= 2
            output "The number, n, must be greater than 2.
                Please re-enter."
            input N
      end loop
      FIB = FIBONNACI(N)
      output "The nth Fibonnaci number is ", FIB
endmain FIBSEQ
```

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M96 HL P2 Q4) (cont.)

A possible solution for (b):

```
function FIBONNACI(val N integer)
result integer

      if N = 1 then
            return 1
      else
            if N = 2 then
                  return 1
            else
                  return FIBONNACI(N - 1) + FIBONNACI(N - 2)
            endif
      end if
endfunction FIBONNACI
```

Binary Search (N96 HL P2 Q1)

(a) To solve a particular computer problem, data has to be stored in sorted order.

If an array is used, a sorted order can be obtained as the data is entered initially, for example, if the input data is:

Grapefruit, Apple, Banana, Pear, Orange, Grape, Kiwi.

If the array to store them is `FRUIT`, after the first read it would store:

Grapefruit			……

After the second read, it would store:

Apple	Grapefruit		……

(i) Draw the array to show how it would look after every subsequent read.

(ii) Draw and describe a data structure which would not require as much data rearrangement, when a new item is added, to maintain a sorted order. Give brief details of its organization.

(b) A binary search can be used to locate a data item in a sorted array of size N. It does this by comparing the middle item of a list with the required input value. If the input is equal to the middle item of the list the search stops, otherwise the half of the list which cannot contain the item is ignored by reassigning the current value for the top or bottom of the list.

For example, using the final array from (a)(i) above, where `N= 7` and the item to find is Kiwi :

initially `BOTTOM` is 1 and `TOP` is `7` (that is, `N`).

Apple	Banana	Grape	Grapefruit	Kiwi	Orange	Pear

Thus `MIDDLE = (TOP + BOTTOM) / 2 = 4`

Since the item at location 4 (Grapefruit) is not equal to the input value (Kiwi) the search does not stop and since Grapefruit is less than Kiwi, the input value cannot be in the lower half of the list, so bottom is reassigned to `MIDDLE + 1` and the search continues.

(i) Continue the trace of the binary search, using a suitable layout.

(ii) Construct the algorithm fragment to perform the binary search. (Remember to give an error message if the item is not found.)

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16 Binary Search (N96 HL P2 Q1)

(c) The binary search can be expressed as a recursive routine.

(i) Explain the term *recursion* and state one necessary condition required by any recursive routine.

(ii) A call to a recursive binary search routine when looking in array `FRUIT` for item `SEEK` could be:

`BSEARCH(SEEK, BOTTOM, TOP,FRUIT).`

Construct the recursive algorithm fragment that would carry out this call. No loop is used, instead the routine is used recursively with suitable parameters.

A possible solution for (b)(ii):

```
procedure BINSEARCH(val SEEK string, val BOTTOM integer,
                    val TOP integer, val FRUIT string array)

        /*  Looking for the occurrence of SEEK within FRUIT */
    declare  MIDDLE integer

    repeat
        MIDDLE <-- (BOTTOM + TOP) div 2
        if FRUIT[MIDDLE] = SEEK then
            output "Found"
        else
            if FRUIT[MIDDLE] > SEEK then
                TOP <-- MIDDLE - 1
            else
                BOTTOM <-- MIDDLE + 1
            endif
        endif
    until (FRUIT[MIDDLE] = SEEK) or (TOP < BOTTOM)

    if TOP < BOTTOM then
        output "Error"
    endif
endprocedure BINSEARCH
```

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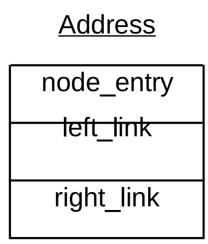
16 Binary Search (N96 HL P2 Q1)

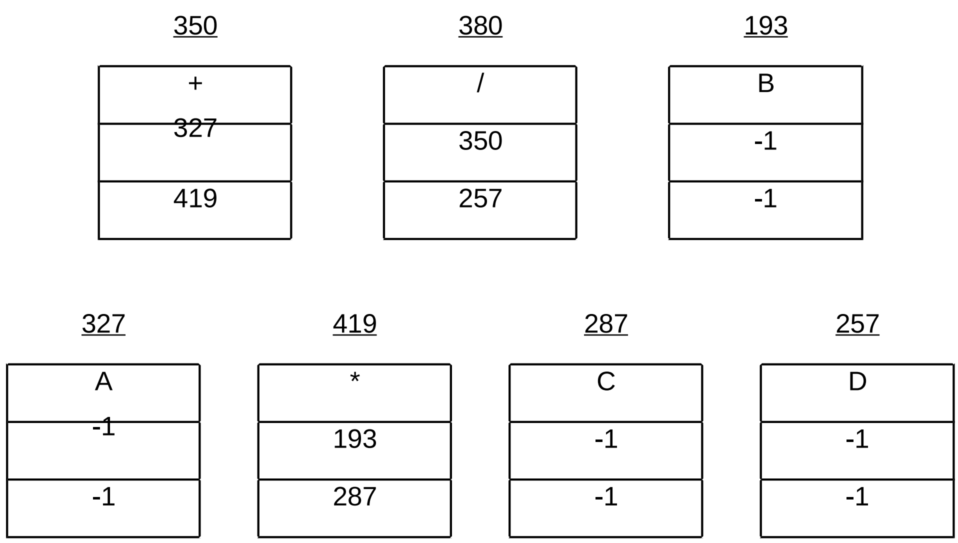
A possible solution for (c) (ii):

```
procedure BSEARCH(val SEEK string, val START integer,
                       val FINISH integer, val FRUIT string array)

/*  Looking for the occurrence of SEEK within FRUIT recursively */

      declare  MIDDLE integer

      if FINISH < START then
            output "Error"
      else
            MIDDLE <-- (START + FINISH) div 2
            if FRUIT[MIDDLE] = SEEK then
                  output "Found"
            else
                  if FRUIT[MIDDLE] > SEEK then
                        BSEARCH(FRUIT, START, MIDDLE - 1)
                  else
                        BSEARCH(FRUIT, MIDDLE + 1, FINISH)
                  endif
            endif
      endif
endprocedure BSEARCH
```

Binary Tree (N96 HL P2 Q7)

A binary tree is stored in a dynamic data structure as follows. Each node is given in the format:





(a) State what the values associated with left_link and right_link represent.

(b) Draw the binary tree presented above if the root node is 380.

estion continues on the following page)

17 Binary Tree (N96 HL P2 Q7) (cont.)

(c) A recursive routine is used to traverse this tree. Assume the new type NODE has been defined as given below.

```
newtype NODE  record
                 NODE_ENTRY character
                 LEFT_LINK  pointer->NODE
                 RIGHT_LINK  pointer->NODE
              endrecord

procedure TRAVERSE(val VALUE pointer->NODE)
      if VALUE->LEFT_LINK # -1 then
            TRAVERSE(VALUE->LEFT_LINK]
      endif
      if  VALUE->RIGHT_LINK # -1 then
            TRAVERSE(VALUE->RIGHT_LINK)
      endif
      output  VALUE->NODE_ENTRY
endprocedure TRAVERSE
```

Trace the algorithm with the initial call TRAVERSE(380).

(d) Give the in-order traversal of the tree given in (b).

(e) State the changes needed to be made to the algorithm in (c) to generate an in-order traversal of a tree.

Summing in a Two-dimensional Array (N97 HL P1 Q10)

The two-dimensional array, called `GRID`, has the following values.

8	16	12	19	4
3	7	15	21	1
2	14	17	5	10
6	13	12	18	9

For example, `GRID[3, 2] = 14.`

Consider the algorithm fragment below:

```
for ROW <-- 1 upto 4 do
      for COL <-- 2 upto 5 do
            if GRID[ROW, COL] < GRID[ROW, COL-1] then
                  GRID[ROW, COL] <-- GRID[ROW, COL-1]
            endif
      endfor
endfor
```

(a) Redraw the array and its new contents **after** tracing the algorithm above.

(b) Construct the algorithm that displays both the average (mean value) of each column and the overall average.

A possible solution for (b):

```
procedure AVERAGE(ref GRID integer array [1..4,1..5])

      declare ROW, COL, COLSUM, ALLSUM  integer
      declare COLAVG, ALLAVG  real

      ALLSUM <-- 0
      for COL <-- 1 upto 5 do
            COLSUM <-- 0
            for ROW <-- 1 upto 4 do
                  COLSUM <-- COLSUM + GRID[ROW, COL]
                  ALLSUM <-- ALLSUM + GRID[ROW, COL]
            endfor
            COLAVG <-- COLSUM / 4
            output "In column ", COL, " the average is ", COLAVG
      endfor
      ALLAVG <-- ALLSUM / 20
      output "The overall average is", ALLAVG
endprocedure AVERAGE
```

Random Numbers (N97 HL P2 Q1)

One method of generating pseudo-random numbers is to start with two values (seeds), multiply them together and then extract the middle digits. This value can then be used as a seed to generate the next value.

For example, if 85 and 65 are used as the seeds, the following is obtained:

Iteration	Seed 1	Seed 2	Product	Pseudo-random Number

1	85	65	5525	52
2	65	52	3380	38
3	52	-	-	-

(a) Complete the table above to show up to iteration 6.

(b) Construct the algorithm `PSEUD` that will calculate and display the first six pseudo-random numbers. You may assume that there is a function `EXTRACT` which returns the middle two digits from a four digit number. For example, `EXTRACT(5525)` would return `52`.

(c) Construct the algorithm for `EXTRACT` as used in part (b). Remember that **`truncate`**`(43.8)` returns `43`.

(d) Explain what would happen if the product was 2008.

(e) Another way to generate pseudo-random numbers is to use modulo arithmetic. For example, in a sequence of values the new number is calculated by multiplying the last generated random value by a set number and using the remainder after dividing by another set value.

For example, with an initial seed of 4, a multiplier of 3, and a modulus of 7, the following is obtained:

Iteration	Pseudo-random Number (R_n)	$3*R_n$	$R_{n+1} = (3*R_n)$ **mod** 7
1	4	12	5
2	5	15	1
3	1	3	3

(i) Construct the recursive algorithm `PSEUD2(ITER, NUM)` which outputs any value in the sequence. For example, the call `PSEUD2(2, 4)` would output 5, where the first parameter represents the sequence number required and the second parameter represents the seed.

(ii) Construct `PSEUD2` as an iterative, rather than a recursive, algorithm.

(iii) State **one** advantage and **one** disadvantage of recursive routines when compared to iterative routines.

(question continues on the following page) **19 Random Numbers (N96 HL P2 Q1)**

A possible solution for (b):

```
main PSEUD
      declare  SEED1, SEED2, X, PRODUCT, RANDOM  integer

      SEED1 <-- 85
      SEED2 <-- 65
      for X <-- 1 upto 6 do
            PRODUCT <-- SEED1 * SEED2
```

```
            RANDOM <-- EXTRACT(PRODUCT)
            output RANDOM
            SEED1 <-- SEED2
            SEED2 <-- RANDOM
      endfor
endmain PSEUD
```

A possible solution for (c):

```
function EXTRACT(val VALUE integer)
result integer
      declare  HUN, LEFT  integer

      VALUE <-- truncate(VALUE / 10)
      HUN <-- truncate(VALUE / 100)
      HUN <-- HUN * 100
      LEFT <-- VALUE - HUN
      return LEFT
endfunction EXTRACT
```

A possible solution for (e) (i):

```
procedure PSEUD2(val ITER integer,val NUM integer)

      if ITER = 1 then
            output NUM
      else
            PSEUD2(ITER - 1, (3 * NUM) mod 7)
      endif
endprocedure PSEUD2
```

uestion continues on the following page)**19 Random Numbers (N96 HL P2 Q1)**

A possible solution for (e) (ii):

```
procedure PSEUD2(val ITER integer,val NUM integer)
      declare  RES, TIMES  integer

      RES <-- NUM
      if ITER > 1 then
            for TIMES <-- 2 upto ITER do
                  RES <--  RES * 3 mod 7
```

```
        endfor
    endif
    output RES
```